

Constraining Lorentz violations with GRB's.

Maria Rodríguez Martínez
Tsvi Piran

Outline :

- Review of LI theory
- Breaking LI
- Constraints from GRB's

Introduction

- Lorentz violating theories have generated a lot of interest.
 - It is important to Test the fundamentals of physics.
- LV is associated with a coherent theory of quantum gravity
 - suppressed by E_p^{-1}
- We only break boost invariance and consider generalizations of the dispersion relation.

Lorentz invariant theory

- The simplest action for a relativistic particle is:

$$S = -mc \int ds = -mc \int \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau$$

proper time

- FRW metric:

$$ds^2 = -c^2 d\tau^2 + a^2(\tau) dx^\mu dx_\mu$$

$$\rightarrow \mathcal{L} = -mc \sqrt{1 - a^2 \frac{\dot{x}^2}{c^2}}$$

- The hamiltonian is:

$$\mathcal{H} = p\dot{x} - \mathcal{L} = \sqrt{c^4 m^2 + \frac{c^2 p^2}{a^2}} = E$$

conjugated momentum:

$$p = \frac{\partial \mathcal{L}}{\partial \dot{x}}$$

energy of a particle traveling in an expanding Universe

• Equations of motion:

$$\dot{p} = -\frac{\partial \mathcal{H}}{\partial x} \quad ; \quad \dot{x} = \frac{\partial \mathcal{H}}{\partial p}$$

$$\rightarrow x(\tau, p) = \int \frac{c^2 p^2}{a^2} \frac{d\tau}{\sqrt{m^2 c^4 + \frac{p^2 c^2}{a^2}}}$$

• Photons : $m=0$

• $a \dot{x} = c \rightarrow$ universal speed of light

• $E = \frac{pc}{a} \rightarrow$ the energy is redshifted

Breaking Lorentz invariance

- We maintain : - E, p conservation
- rotation and translation invariance

- . We break boost invariance

→ there is a preferred frame (CMB)

- . New dispersion relation:

first term of
an exp. in ξ^{-1}

$$E^2 = m^2 c^4 + \frac{p^2 c^2}{a^2} \left(1 + \xi \left(\frac{cp}{\xi E_p a} \right)^n + \dots \right)$$

. $\xi = \pm 1$

. ξE_p : scale sym. breaking

- . The speed of light is p-dependent

$$v \approx c \left(1 + \frac{\xi}{2} (1+n) \left(\frac{cp}{\xi E_p a} \right)^n \right)$$

From the dispersion relation we deduce the hamiltonian and the eq. of motion

Assume FRW : $H(z)^2 = H_0^2 \left(\Omega_m (1+z)^3 + \Omega_\Lambda \right)$

$$\rightarrow a_0 \times (z, p) = \frac{1}{H_0} \int_0^z \left[1 + \varepsilon \frac{Hn}{2} \frac{\mu_0^n}{\xi^n} (1+\xi)^n \right] \frac{d\xi}{(\Omega_m (1+\xi)^3 + \Omega_\Lambda)^{y_2}}$$

$$\cdot \mu(a) = \frac{pc}{E_p a} : \text{photon energy-Planck energy ratio}$$

$\rightarrow a_0 \dot{x} = f(p) : \text{the speed of light depends on } p$

$\rightarrow x(z, p)$ is not a geodesic of the LI action

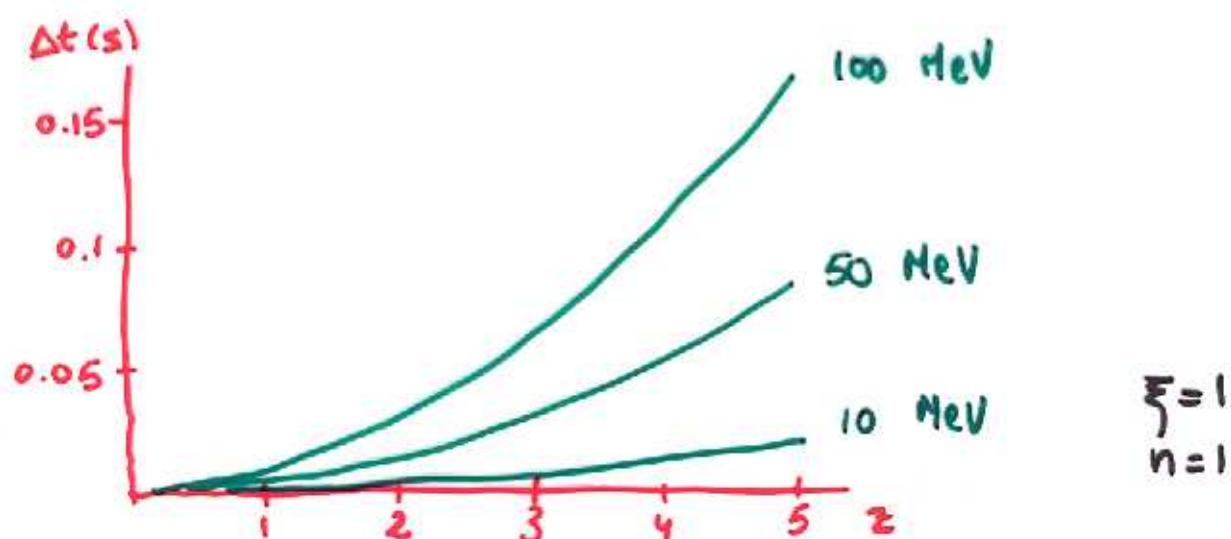
$$\Delta t = t(\mu) - t_{LI}$$

time a γ at
 $t(\mu)$ takes to
 reach the Earth

time a γ at
 c takes to reach
 the Earth

$$\Delta t = \epsilon \frac{1+n}{2H_0} \frac{\mu_0^n}{\xi^n} \left(\frac{\Omega_m (1+z)^3 + \Omega_\Lambda}{\Omega_m + \Omega_\Lambda} \right)^{Y_2} \int_0^z \frac{(1+z)^n dz}{\left(\Omega_m (1+z)^3 + \Omega_\Lambda \right)^{Y_2}}$$

$$\mu_0 = \frac{c_p}{\epsilon_p a_0}$$



Newtonian analysis

$$\cdot \Delta t \sim \frac{L}{v} - \frac{L}{c} \quad ; \quad \epsilon^2 = \frac{P^2 c^2}{a^2} \left(1 + \epsilon \frac{\mu^n}{\xi^n} + \dots \right)$$

group velocity $\leftarrow \cdot v = \frac{dE}{d(P/a)} \approx c \left(1 + \frac{\epsilon}{2}(n+1) \frac{\mu^n}{\xi^n} + \dots \right)$

$$\cdot L \sim \frac{c}{u_0} \frac{z}{\sqrt{\Omega_m + \Omega_v}} \quad , \quad z < 1$$

$$\rightarrow \Delta t \sim \epsilon \frac{1+n}{2u_0} \frac{z}{\sqrt{\Omega_m + \Omega_n}} \frac{\mu_0^n}{\xi^n}$$

→ this is the first order expansion
of the relativistic expression.

Testing LV with GRB's

• High energy spectrum of a GRB:

$$N(\epsilon) = R, \epsilon^{-\beta} \quad , \quad \epsilon \geq \epsilon_0 \sim 100 \text{ KeV}$$

• # of photons detected in (ϵ_1, ϵ_2) with a telescope A during the burst:

$$N(\epsilon_1, \epsilon_2) = \frac{A R_1}{4\pi d^2(z)} \int_{\epsilon_1(1+z)}^{\epsilon_2(1+z)} \epsilon^{-\beta} d\epsilon$$

• $d(z)$ is the cosmological distance:

$$d(z) = \frac{c}{H_0} \sqrt{\int_0^z \frac{dz}{(\Omega_m(1+z)^3 + \Omega_\Lambda)^{1/2}}}$$

* notation

- $\epsilon \sim$ rest-frame GRB
- $E \sim$ Earth-frame

R_i is related to the bolometric isotropic energy

$$E_{\gamma, \text{iso}} = R_i \int_{E_0}^{\infty} \epsilon^{-\beta} \epsilon d\epsilon$$

Time resolution of the telescope :

$$\Delta t(E_1, E_2) \sim \frac{1}{\sqrt{(E_1, E_2)/T}} =$$

$$= \frac{4\pi d^2(z)}{A} \frac{\beta-1}{\beta-2} \frac{T}{E_{\gamma, \text{iso}}} (1+z)^{\beta-1} \frac{\epsilon_0^{2-\beta}}{\epsilon_1^{1-\beta} - \epsilon_2^{1-\beta}}$$

$$\frac{E_{\gamma, \text{iso}}}{T} \rightarrow \tau_{\text{peak}} : \text{maximum time resolution}$$

Setting a bound on ξ

- No LV has been observed so far:

$$(\Delta t)_{\text{delay}} < (\Delta t)_{\text{resolution}}$$

$$\Rightarrow \xi^2 > \frac{A H_0}{8\pi c^2} \frac{\beta-2}{\beta-1} \xi_0^{\beta-2} \cancel{\propto} \left(1 - \left(\frac{E_2}{E_1}\right)^{1-\beta}\right) \frac{\mu_0^n}{E_1^{\beta-1}} \cdot G_n(z)$$

↑ luminosity ↑ energy band ↑ redshift

- Energy band dependence

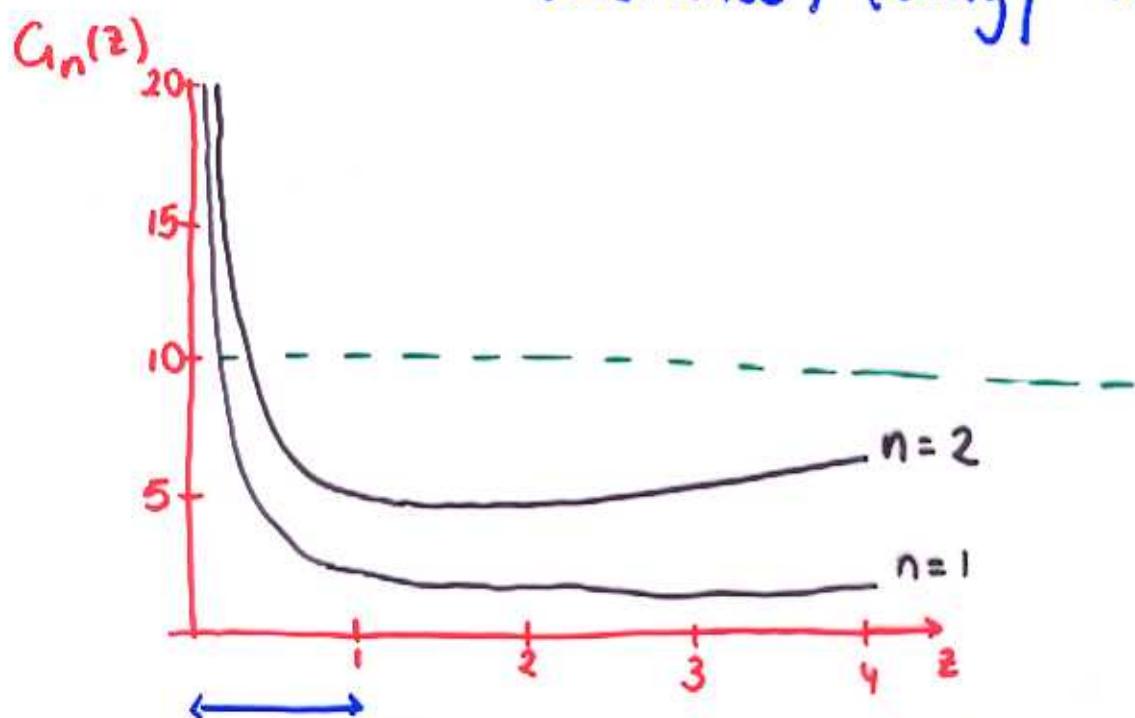
$$\frac{\mu_0^n}{E_1^{\beta-1}} > \frac{E_1^{n-\beta+1}}{E_{\text{pl}}^n} \quad \left(\mu_0 = \frac{cP}{E_p \alpha_0} \right)$$

- ($n \geq 2$) a) $n - \beta + 1 > 0$: preferable high energies
- ($n=1$) b) $n - \beta + 1 < 0$: low energies is better

- We keep (E_1, E_2) small to avoid lack of simultaneity problems.

Redshift dependence

$$G_n(z) = \frac{\text{delay } (z)}{(\text{distance})^2 (\text{energy redshift})}$$



close bursts :

- many γ detected
- $\Delta t_{\text{res}} \uparrow$

far bursts :

- $\Delta t_{\text{del}} \uparrow$
- few γ detected

→ It is better to look at close bursts.

A quantitative example : GRB 050603

$$\xi^n > \sigma \frac{\mathcal{L}_P}{\mathcal{L}_*} C_n(z) \quad \mathcal{L}_* = 5 \cdot 10^{52} \frac{\text{erg}}{\text{s}}$$

• GRB 050603 : $z = 2.821$; $\beta = 2.15$

• Konus-Wind : $A \sim 200 \text{ cm}^2$

$$P_{\text{peak}} (20 \text{ keV}, 3 \text{ MeV}) = 3.2 \cdot 10^{-5} \text{ erg/s.cm}^2$$

$$\rightarrow n=1 : \sigma = 0.0086, \quad \boxed{\xi_1 > 0.3}$$

$$\rightarrow n=2 : \sigma = 10^{-26}, \quad \boxed{\xi_2 > 10^{-12}}$$

• Swift-BAT : $A \sim 5200 \text{ cm}^2$

$$P_{\text{peak}} (15 \text{ keV}, 350 \text{ keV}) = 31.8 \text{ }\sigma/\text{cm}^2 \cdot \text{s}$$

$$\cdot n=1 : \xi_1 > 1.08$$

$$\cdot n=2 : \xi_2 > 2 \cdot 10^{-12}$$

Conclusions

- GRB's are powerful tools to test high energy physics.

! · for $n=1$, we investigate $\sim E_p$

· however:

- telescope's time resolution:

$$(\Delta t)_{\text{theoretical}} \leftrightarrow (\Delta t)_{\text{detector}}$$

- we need at least 2 energy channels

- simultaneity